CURVELET DOMAIN IMAGE FUSION OF OCT AND FUNDUS IMAGERY USING CONVOLUTION OF MERIDIAN DISTRIBUTIONS

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ABSTRACT

This paper presents a novel, statistical model based method aimed at fusing Optical Coherence Tomography and Fundus Photographic imagery of the eye. The presented method utilizes the Discrete Curvelet Transform to decompose the images into sub-band coefficients. The Meridian distribution, a specialized case of the generalized Cauchy distribution, is used to model the curvelet decomposition coefficients. The convolution of the input image distributions is used as a probabilistic prior for modelling the fused image coefficients. Experimental results show this method to provide very high-quality fusion results.

Index Terms— Image fusion, OCT, Fundus Photography, Meridian distribution, Curvelets.

1. INTRODUCTION

An application of medical imaging is in obtaining imagery of the human eye as this can provide information instrumental to the diagnosis and treatment of numerous diseases and conditions. Fundus photography is a common method for obtaining images of the eye, as it effectively allows for the acquisition of photographs of the retina. Another method recently gaining in popularity is that of Optical Coherence Tomography (OCT). In contrast to many other methods, OCT can capture very high (micrometer) resolution 3D images of the retina. This ability of the OCT to penetrate the scattering medium to a certain depth makes it a very valuable tool to ophthalmologists as it shows the sub-surface portion of the eye and can provide complementary information unseen in a fundus photograph. Due to this property OCT is often a very desirable tool in diagnosing various eye conditions such as retinopathy and a plenitude of macular disorders and disorders of the optical nerve [1, 12].

Unfortunately OCT images may suffer from “misalignments” of data slices that appear as black areas devoid of data, caused by involuntary eye movements during the scanning process. Fusing OCT and Fundus images allows for a more concise overview of the human eye than provided by any of the methods alone.

To this effect, many fusion methods have been proposed in recent years, aiming to combine the advantages of two different imaging modalities. These methods can be distinguished into various categories, such as transform based methods (most often utilizing the wavelet transform) [3], pixel- and ICA-based methods [13].

In the past few years wavelets have been criticized for their somewhat limited ability to sparsely model sharp discontinuities in 2-dimensional data [4, 5]. New transforms such as the ridgelet and curvelet transforms have been proposed, aiming to surpass wavelets in image processing by addressing this problem. Curvelets are a highly anisotropic multiscale architecture for the representation of 2-dimensional signals that are ideally adapted to representing objects that display curve-punctuated smoothness – that is, smoothness except for a discontinuity along a general curve with bounded curvature [5]. As they offer increased sparsity of representation, curvelets are able to represent a smooth contour using far fewer coefficients than wavelets, for the same accuracy [6]. This makes curvelets especially suitable for image processing and is an indication that many wavelet based fusion methods could benefit from being applied in the curvelet domain.

Our proposed method makes use of the Meridian distribution to model the distribution of the curvelet decomposition coefficients. Previous work [6] has shown that the distribution of the curvelet decomposition coefficients of an image exhibits a highly non-Gaussian behaviour. The distribution is of a leptokurtic nature, with

![Flowchart of the proposed fusion process](Image)
very heavy tails. The Meridian distribution, a specialized case of the Generalised Cauchy Distribution has been shown to be a very good model for coefficients exhibiting this statistical behaviour and has been successfully used to model wavelet coefficients with similar properties by Agrawal et al. [2].

The coefficient weight pairs are optimized by means of Maximum Likelihood estimation as shown in [3], a method that generally presents higher quality results than the classic Weighted Average presented in 1993 by Burt and Kolczynski [7]. This method introduces a degree of optimality in the fusion process as it removes the ad-hoc match & saliency measure thresholds used in WA – based methods. The presented method seeks to apply the Meridian Convolution process in the Curvelet Domain, achieving superior results in comparison to Wavelet implementations.

The paper is organized as follows: In Section 2, we provide a brief overview of the mathematical properties of the Meridian distribution. Section 3 describes the curvelet-domain fusion algorithm while Section 4 discusses results obtained using the proposed method on real retinal images. Finally, Section 5 provides a brief conclusion of the paper. Fused images created using the proposed method can be found at the end of the paper.

2. MERIDIAN DISTRIBUTION

The Meridian distribution is a specific case of the Generalized Cauchy Distribution. The GCD, as used here, is taken to have the following density function, as shown in [2]

\[ f(x) = a \gamma (|x|^p + |x|)^{-\frac{\gamma p}{p}} \] (1)

where \( \gamma \) and \( p \) are taken as the scale parameter and tail constant respectively. Additionally, \( \alpha \) is taken as

\[ \frac{p \Gamma(2/p)}{2(\Gamma(1/p))^2} \] (2)

For this family of distributions, fixing the tail constant \( p \) to 1 provides us with the Meridian Distribution whereas a \( p \) equal to 2 results in the Cauchy Distribution. Decreasing the value of \( p=2 \) from the Cauchy case results in an even slower tail decay, further accentuating the heavy-tailed nature of the distribution. As a result we propose that the Meridian Distribution will be better suited at the modelling of extremely heavy tailed curvelet coefficients than the simple Cauchy case.

2.1. Parameter estimation

Parameter estimation for the GCD, and for the Meridian Distribution in particular is achieved using the Mellin Transform-based [8] method presented in [2]. We reiterate here that for a function \( f \), defined over \( \mathbb{R}_+ \), its Mellin Transform is defined as

\[ \Phi(z) = M[f(u)](z) = \int_0^{\infty} u^{z-1} f(u) \, du \] (3)

Where \( z \) is the complex variable of the transform.

As the Mellin transform bears great similarities to the Fourier Transform, we can draw analogies in the way various statistical measures can be deducted from each transform. Based on this observation, we can show that the \( r^{th} \) - order second-kind cumulants of the Mellin Transform are

\[ \hat{k}_r = \frac{d^r \Phi(z)}{dz^r} \bigg|_{z=1} \] (4)

And \( \Phi(z) = \log (\Phi(z)) \). The first two second-kind cumulants are then empirically derived from a number N of samples \( x_i \) as follows

\[ \hat{k}_1 = \frac{1}{N} \sum_{i=1}^{N} \log(x_i) \] (5)

\[ \hat{k}_2 = \frac{1}{N} \sum_{i=1}^{N} [\log(x_i) - \hat{k}_1]^2 \] (6)

Further manipulation of equations (3) and (1) can provide the formula for the second-kind characteristic function of the GCD density:

\[ \Phi(z) = \frac{\gamma \left( \frac{2}{p} \right)^{\frac{1}{p^2}} \Gamma \left( \frac{1}{p^2} \right)^{\frac{1}{p}}}{4 \Gamma \left( \frac{1}{p^2} \right)^{\frac{1}{p}} \Gamma \left( \frac{2}{p} \right)^{\frac{1}{p}} z^2} \] (7)

The cumulants can now be shown to be [2]

\[ \hat{k}_1 = \log \gamma \] (8)

\[ \hat{k}_2 = \frac{2 \psi(1/2)}{p^2} \] (9)

where we set \( \psi(t) = \frac{d^{n+1}}{dt^{n+1}} \log \Gamma(t) \) as the poly-gamma function and we deduce \( \gamma \) by solving (8) and (5).

3. FUSION SCHEME

3.1. Sparsity of Representation

An important notion for many image processing applications, including image fusion, is that of sparsity in the mathematical representation of an image. A sparse mathematical decomposition of an image would suggest a very large number of coefficients with very small amplitudes and a small number of coefficients with large amplitudes. This further highlights the idea of saliency, as it becomes easy to identify the salient coefficients, i.e. the ones which contribute heaviest to the composition of the image. Looking at the task of image fusion as an
optimization process, it would be expected that the product of a fusion process should enhance the sparsity inherent in the input images by giving more emphasis (heavier averaging weights) to the most salient coefficients.

The sparsity of wavelet decompositions of natural images has been one of the major reasons for their widespread usage in the area of signal processing – the curvelet transform used here surpasses the wavelet transform in this property by allowing for sparser edge representation, resulting in further overall sparsity in image coefficient data [5]. This enhanced sparsity is one of the properties making the Curvelet Transform a most compelling signal processing transform.

3.2. Convolution of Meridian Distributions

The fusion method presented in this paper belongs to the weighted averaging family of fusion rules – the input curvelet coefficients \( C_1, C_2 \) are averaged using different weights \( w_1, w_2 \) resulting in the fused coefficient \( C_f \). For the work presented here we assume that the curvelet decomposition coefficients are distributed according to a Meridian distribution. Since the fused coefficients are a linear sum of the input coefficients, it can be shown that if the input coefficients are indeed distributed along a Meridian distribution, the fused coefficients will also follow a Meridian distribution. In particular, the PDF for the Meridian distribution is [9]

\[
P(x; \mu, \gamma) = \frac{\gamma}{2((x-\mu)+\gamma)^2}
\]

where \( \mu \) is the location parameter and \( \gamma (\gamma > 0) \) is the scale parameter of the distribution – in effect \( \mu \) specifies the center peak of the distribution and \( \gamma \) the spread.

Now in the case of a convolution of two meridian distributions, we obtain

\[
P(y; \mu, \gamma) = P(x; \mu_1, \gamma_1) \cdot P(x; \mu_2, \gamma_2)
\]

where \( \mu \) is the location parameter and \( \gamma (\gamma > 0) \) is the scale parameter of the distribution – in effect \( \mu \) specifies the center peak of the distribution and \( \gamma \) the spread.

3.3. Weights Optimisation

Our proposed method relies on the maximization of the cost function obtained from the PDF of the convoluted Meridian distributions of the input images, which is obtained as:

\[
P(x; \mu, \gamma) = \frac{\gamma_1 + \gamma_2}{2((x-(\mu_1+\mu_2))+(w_1\gamma_1+w_2\gamma_2))^2}
\]

Maximum Likelihood is used to optimize the fusion coefficient weights – it is performed by maximizing the cost function of the likelihood formula:

\[
L = -\log(P(x; \mu, \gamma))
\]

\[
C(w_1, w_2) = E[L]
\]

The set of weights that maximizes (16) is taken as the set of optimal weights. As with all fusion schemes making use of weighted averages, the weights are positive and must sum up to one in all cases. The partial derivatives of the cost function with respect to the input weights are

\[
\frac{\partial C(w_1, w_2)}{\partial w_1} = E \left\{ -\frac{\gamma_2}{\gamma} + \frac{2(x_1 - \mu_1) + 2\gamma_1 y_1 w_1 + 2\gamma_2 y_2 w_2 + Q_1}{(x-\mu)^2 + \gamma^2} \right\}
\]

\[
\frac{\partial C(w_1, w_2)}{\partial w_2} = E \left\{ -\frac{\gamma_2}{\gamma} + \frac{2(x_2 - \mu_2) + 2\gamma_1 y_1 w_1 + 2\gamma_2 y_2 w_2 + Q_2}{(x-\mu)^2 + \gamma^2} \right\}
\]

where the coefficients, weights and statistical parameters are as defined above. As in [2] the chosen implementation method for MLE is that of steepest ascent with update procedures given by

\[
W_{1,k+1} = w_{1,k} + \eta \frac{\partial C(w_1, w_2)}{\partial w_1} \]

\[
W_{2,k+1} = w_{2,k} + \eta \frac{\partial C(w_1, w_2)}{\partial w_2}
\]

where \( \eta \) is the learning rate of the update process and \( k \) is the iterative index. For the work presented in this paper this learning rate \( \eta \) has been kept at 0.05. This update process is iterated until a certain error criterion is met, namely

\[
\left| W_{1,k+1} - W_{1,k} \right| \leq \varepsilon
\]

And \( \varepsilon \) is the selected error threshold, in this case set at 0.001. Increasing the value of the learning rate or the error threshold may allow for faster computation but can severely
limit the quality of the fusion result, hence why both variables are kept at such low values.

4. RESULTS

The proposed fusion method was evaluated both using image fusion metrics and on the basis of visual perception. The usage of various image fusion metrics is perhaps the only available method to objectively assess the performance of a given fusion rule. In absence of a ground truth or reference image to compare results against, we resort to the usage of the Piella and Petrovic image fusion metrics [10, 11]. While these metrics provide a generally reliable and quantitative assessment of the quality of fused images, their assessment does not always correlate very well with the quality of an image as perceived by a human observer. It is therefore of paramount importance not to blindly accept metric results before comparing and contrasting them with a subjective, visual test.

In the area of medical imaging, fusion rules should aim to retain all salient features in the input images, such as edges and changes in contrast & coloration as these features are critical in the process of medical diagnosis and treatment. The introduction of any artifacts can be most detrimental as it can hinder the detection of any abnormalities or deterioration in various parts of the optical tissue and vessel network system.

Experimental results have shown our method to outperform its wavelet counterparts. The images fused using the proposed method appear to retain all their salient features – the vessel structures are clearly defined. No artefacts are introduced and some present in wavelet implementations have disappeared. Besides visual evaluation of the data, objective evaluation by means of the aforementioned metrics validates these results, as shown in the table below.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Wavelet Weighted Averaging</th>
<th>Wavelet Meridian Convolution</th>
<th>Curvelet Meridian Convolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piella</td>
<td>0.6146</td>
<td>0.6996</td>
<td>0.7245</td>
</tr>
<tr>
<td>Petrovic</td>
<td>0.4891</td>
<td>0.5288</td>
<td>0.5385</td>
</tr>
</tbody>
</table>

Figure 2: Metric Performance Comparison

5. CONCLUSION

This paper has sought to apply the Meridian distribution to the modelling of curvelet coefficients and consequently use it in performing statistical convolution fusion of OCT and Fundus Photographic medical imagery. The results show that the distribution is indeed a suitable model for image curvelet coefficients and the method provides high quality fused images, outperforming other methods.

6. REFERENCES


